ISSN 2229-5518

BER for Uniformly Quantized and Non-Uniform Quantized Synchronous CDMA Signals in AWGN Channels with MMSE Multiuser Receiver

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Abstract—This research work analyzed the performance of minimum mean-square error (MMSE) multiuser receiver for both uniformly quantized and unquantized synchronous code division multiple access (CDMA) signals in additive white Gaussian noise (AWGN) channels. This project is mainly based on the representation of uniform quantizer by gain plus additive noise model. Based on this model, we derived the weight vector and the output signal-to-interference ratio (SIR) of the MMSE receiver. The effects of quantization on the MMSE receiver performance is characterized in a single parameter named "equivalent noise variance". The optimal quantizer step size which maximizes the MMSE receiver output SNR is also determined.

Index term-Code division multiacess, signal detection,quantization, additive white Gaussian noise channel.

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1 INTRODUCTION

Code Division Multiple Access (spread spectrum): a unique code is assigned to each user. This code is used to 'code' the data message. As codes are selected for the cross correlation properties, all users can transmit simultaneously in the same frequency channel while a receiver is still capable of recovering the desired signal. Synchronization between links is not strictly and so random access is possible. A practical application at the moment is the cellular CDMA phone system.

In DS/CDMA system,input data is directly modulated using the PN sequences that the resulting signal has the same bandwidth as the rate of the spreading (PN) sequence.After modulation,AWGN adds to the transmitted signal and it is passed through the channel.

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At the front end of the receiver,again AWGN adds to the signal.So to eliminate the effects of noise, matched filter is used at the receiver.This matched filter removes the noise and it extracts the original signal that was transmitted.In demodulation,again PN sequence is used to despread the signal so that the original(i/p) data is extracted at the output.

In the case of multiple users, each user has separate spreading code. To extract the information from a particular user, the receiver has to know the code sequence of that particular user. So for detecting multiple users, a bank of matched filters is used. Each matched filter has the spreading code of corresponding users signal.

The spreaded signals of all the users are combined either at the base station transmitter or in the channel and then they are transmitted at a time through the same channel. To detect a particular user's signal , it is corresponding matched filter accepts only that signal and ignores all other signals from other users or it considers them as noise. Then that signal despreaded to get original data.

2 System Model

2.1 Recevied Signal

The model presented here is the discrete-time synchronous CDMA model [1]. Consider a CDMA multiple-access system with N-users, with the k^{th} user transmitting a bit-stream b_k over an AWGN channel. Let the k^{th} user be assigned a signature sequence $s_k[n]$ and let the received amplitude of the k^{th} user be A_k. The combined received signal from the channel is given by the sum:

$$y(t) = \sum_{k=1}^{M} A_k b_k s_k(n) + \sigma \aleph(n)$$

Where,

- b_k 2 { 1, _ 1 } is the input bit-vector corresponding to the k^{th} user.
- A_k is the received amplitude of the k^{th} user.
- matrimonial (n) is a white-gaussian noise sequence with unit PSD.
- $\sigma$is the standard-deviation of the noise present in the channel.

Note that, a one-shot model is used, since only one bit duration is considered. This one-shot model is enough for analysis if synchronous CDMA is considered. Let the normalized cross-correlations of the signature waveforms be defined as:

$$\rho_{ij} = < s_i, s_j > = \sum_{l=1}^{N} s_i(l) s_j(l)$$

Where, N is the length of the signature sequence. Here, the signature waveforms are selected such that $\varrho_{ii} = 1$ (so that the cross-correlations are normalized). Further, we can define a cross-correlation matrix R as:

$$R = \begin{pmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1M} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2M} \\ . & . & . & . \\ . & . & . & . \\ \rho_{M1} & . & . & \rho_{MM} \end{pmatrix}$$

The R-matrix has the following properties:

- It is symmetric.
- The diagonal elements are equal to 1 (normalized)) It is toeplitz.
- In general, the matrix is non-negative definite.

The Matched Filter: The matched-filter in digital communication system is used to generate sufficient statistics for signal detection. In the case of a multiple-access system, the receiver consists of a bank of matched-filter each matched to the corresponding users signature waveforms, as shown in fig (1). The output of the j th correlator is given by:

$$y_j = \sum_{n=1}^{N} y(n) s_j(n)$$

When expanded, the above equation becomes:

$$y_j = \sum_{k=1}^{M} A_k b_k \left(\sum_{n=1}^{N} s_k(n) s_j(n) \right) + \sum_{n=1}^{N} \aleph(n) s_j(n)$$
$$= \sum_{k=1}^{M} A_k b_k \rho_{jk} + n_j$$

The above expression can be written in a compact matrix notation form:

$$y_j = \mathbf{r}_j \mathbf{A} \mathbf{b} + n_j$$

Where,

- $r_j = [\varrho_j1, \varrho_j2, \ldots, \varrho_{jM}]T$, the cross-correlation vector of the j th -user with all the other users.
- $A = diag(A_1, \ldots, A_m)$, the matrix of received signal amplitudes.
- $b = [b_1, \ldots, b_M]T$, the vector of the received bits.

If the outputs of all the users are stacked up, we get:

$$\begin{pmatrix} y_1 \\ . \\ . \\ . \\ y_M \end{pmatrix} = \begin{pmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1M} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2M} \\ . & . & . & . \\ . & . & . & . \\ \rho_{M1} & . & . & \rho_{MM} \end{pmatrix} \begin{pmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & \cdots & 0 \\ . & . & . & 0 \\ . & . & . & . \\ 0 & . & . A_M & \end{pmatrix} \begin{pmatrix} b_1 \\ . \\ . \\ . \\ b_M \end{pmatrix} + \begin{pmatrix} n_1 \\ . \\ . \\ . \\ n_M \end{pmatrix}$$

In compact matrix notation, the above equation can be represented as:

y = RAb+n

The noise at the output of the matched filter, n can be statistically characterized as follows:

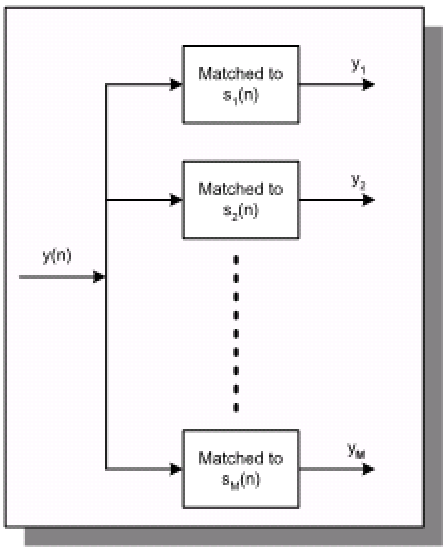

FIGURE 1. The Matched Filter

$$E\{n_i n_j\} = E\left\{\sum_{n=1}^{N} \sigma\aleph(n)s_i(n)\sum_{l=1}^{N}\sigma\aleph(l)s_i(l)\right\}$$

Further simplifying, we get:

$$E\{n_i n_j\} = \sigma^2 E\left\{(s_i(1),\ldots,s_i(N))\begin{pmatrix}\aleph(1)\\ .\\ .\\ \aleph(N)\end{pmatrix}(\aleph(1),\ldots,\aleph(N))\begin{pmatrix}s_j(1)\\ .\\ .\\ s_j(N)\end{pmatrix}\right\}$$

Using the fact that the input noise is a unit-variance white gaussian random variable, the correlation matrix of the input noise will be an identity matrix. Thus, the above equation simplifies to:

$$E\{n_i n_j\} = \sigma^2\sum_{n=1}^{N} s_i(n)s_j(n) = \frac{\sigma^2}{N^2}\rho_{ij}$$

And, the covariance matrix of the output of the matched-filter is thus given by:

$$E\{\mathbf{n}\mathbf{n}^T\} = [E\{n_i n_j\}]_{(i,j)} = \sigma^2\mathbf{R}$$

Using this model (eqn. (9)) developed in this section , the problem of optimal multi-user detection is presented in the next section.

2.2 The Maximum-Likelihood Criterion.

It is a well-known fact [4] that, in the case of detecting signals corrupted by additive white Gaussian noise (AWGN), the decoder that minimizes the probability of error is the Maximum-Likelihood Decoder. The ML criterion is based on selecting the input bit that minimizes the Euclidean distance between the transmitted symbol (corresponding to the input bit) and the received symbol. In the case of multi-user detection, the Euclidean distance between a transmitted symbol vector corresponding to the input bit-vector b and the received symbol vector is given by [1]:

$$d(\mathbf{b}) = \sum_{n=1}^{N}\left[y(n) - \sum_{k=1}^{M} A_k b_k s_k(n)\right]^2$$

Expanding the above expression, we get:

$$d(\mathbf{b}) = \sum_{n=1}^{N} y(n)^2 - 2\sum_{k=1}^{M} A_k b_k \sum_{n=1}^{N} y(n)s_k(n) + \sum_{n=1}^{N}\left(\sum_{k=1}^{M} A_k b_k s_k(n)\right)^2$$

The first term in the expression is independent of b and so it can be removed from the minimization process (instead we define a likelihood function (b) that differs from d(b) by a constant). Using the definitions of y_j in equation (4) and using the definitions of A and b, the above expression can be simplified as:

$$\Omega(\mathbf{b}) = -2N\mathbf{b}^T\mathbf{Ay} + N\mathbf{b}^T\mathbf{ARAb}$$

Again, removing the common factor N and using the fact that maximizing the negative of a function is same as minimizing the function, the problem of optimal multiuser detection can be stated as:

$$\text{Maximize}\ ,\ \Omega(\mathbf{b}) = 2\mathbf{b}^T\mathbf{Ay} - \mathbf{b}^T\mathbf{ARAb}$$
$$\text{Subject to}\ ,\ \mathbf{b}\in\{+1,-1\}^M$$

The maximization problem stated above is a combinatorial optimization problem, since the variables of the optimization problem are basically limited to a finite set. The straight-forward method for solving such combinatorial optimization problem is an exhaustive search over all the possibilities In the above case, since b 2 { +1, _ 1 } M , there are 2 M possibilities. (For Q-ary modulation, have Q N possibilities!). Thus the search space increases in a geometric fashion with the number of users. In other words, the complexity required for decoding M bits of data is O(2 M). It has been shown by Verdu [1] that no-other algorithm whose computational complexity is a polynomial in the number of users exists to solve this combinatorial optimization problem.

3 MMSE Multiuser Receiver for Uniformly Quantized Synchronous CDMA Signals in AWGN channels.

3.1 System Model

Representation of received signal:

In this system we assume that K synchronous users are transmitting CDMA signals in an AWGN channel. The received continuous signal is sent to a matched filter whose output is sampled at the chip rate. An automatic gain control is used to bring the amplitude of the received signal into the range of the quantizer. The received signal vector,r

$$r = \propto_A (SAb + n)$$

In the above equation α_A is the gain offered by AGC, S is the spread spectrum matrix, A is a diagonal matrix of amplitudes, b is the bit sequence that is to be transmitted and n is the white Gaussian noise vector. Each element of this noise vector is gaussian with zero mean and variance σ^2 . S consists of K spreading sequences each of length N, where N is known as the processing gain. $E_1, E_2, \ldots, E_K$ are the bit energies which are bounded by the Lindeberg central limit theorem .Therefore the received signal, r converges to a random vector whose distribution is Gaussian with zero mean. The distribution of r can be normalized to get unity variance by properly selecting α_A .

For normalization $\propto_A$ can be selected as follows. After normalization the distribution of r is $N(0,1)$.

$$\propto_A{}^2 = \left(\frac{1}{N}\sum_1^K E_k + \sigma^2\right)$$

3.1.1 Uniform Quantizer

Now the received signal is quantized using a uniform quantizer. Whenever the amplitude of the received signal falls within x_k and x_{k+1} the signal is approximated to be y_k.

$y_k = q(r)$, if $r \in (x_k, x_{k+1}]$, k = 1, 2,…., L

Where q ( ) is the quantization function and L = 2^R, R is the number of bits used to represent the output of the quantizer and y_k are the representation levels and x_k are the decision thresholds. For a uniform quantizer the representation levels and thresholds are given by

$$x_k = \left[k - \frac{L+2}{2}\right]\Delta, \quad k = 2,3,\ldots L$$

$$y_k = \left[k - \frac{L+1}{2}\right]\Delta, \quad k = 1, 2, 3\ldots..L$$

Where Δ is the step size. $x_1 = -\infty$ And $x_{L+1} = \infty$

3.1.2 Gain Plus Additive Noise Model

To simplify the complexity in determining the weight vector of the MMSE detector we represent the output of the quantizer using gain plus additive noise model. Let z be the output of the quantizer then it can be represented as

$Z = \alpha_g r + \eta$

Where α_g is less than unity gain component and η is the additive noise component with zero mean.

$$\eta = [\eta_1, \eta_2, \ldots\ldots\eta_N]$$

$$\alpha_g = \frac{\sum_1^L y_k \left[e^{\frac{-x_k{}^2}{2}} - e^{\frac{-x_{k+1}{}^2}{2}}\right]}{\sqrt{2\pi}}$$

$$E[\eta_i{}^2] = \sum_1^L y_k{}^2 \left[Q(x_k) - Q(x_{k+1})\right] - \alpha_g{}^2$$

3.1.3 MMSE Multiuser Detector

Now the weight vector of the MMSE multiuser detector is given by the equation $w_k = R^{-1}p_k$ Where

$$R = (\alpha_g \propto_A)^2 (SAAS^T + \sigma^2 I) + \sigma_\eta{}^2 I$$

$$p_k = \alpha_g \propto_A s_k \sqrt{E_k}$$

For user k the MMSE detector output can be computed using the following equation

$m_k = w_k{}^T z$ where m_k is the output of the MMSE detector. m_k can be expressed as

$$m_k = \mu_k b_k + \varepsilon_k$$

Where $\mu_k = p_k{}^T R^{-1} p_k$ represents the amplitude of the signal and ε_k represents the interference.

$E(\varepsilon_k) = 0$

$Var(\varepsilon_k) = \mu_k(1 - \mu_k)$

3.1.4 SIR and BER

Now the SIR, denoted by γ_k can be computed using the following equation.

$$\gamma_k = \frac{\mu_k}{(1-\mu_k)}$$

Since b_k is BPSK modulated its BER is related to the SIR through the equation,

$BER = Q(\sqrt{\gamma_k})$

3.1.5 Equivalent noise variance

Equivalent noise variance is the newly defined quantity which is used to characterize the effects of the quantization on the MMSE detector. It is a function of the sum of each active user's signal-to-noise ratio (SNR), processing gain, and the number of quantization levels. It is denoted by ϵ^2 and is given by the equation

$$\epsilon^2 = \sigma^2 + \frac{\sigma_\eta{}^2}{\left((\alpha_g \propto_A)^2\right)} i$$

3.1.6 Optimum uniform quantizer

The optimum uniform quantizer in the sense of minimizing the BER or maximizing the receiver output SIR is the one maximizing the value of μ_k, which is equivalent to minimizing the equivalent noise variance. The optimum step size can be calculated by minimizing the following equation using standard optimization softwares.

$$\Delta_0 = \operatorname{argmin}\left(\sigma^2 + \frac{\sigma_\eta{}^2}{\left((\alpha_g \propto_A)^2\right)}\right)$$

4 OBSERVATIONS

We plot the curves for N=64 and K=32.We assume that the received power is constant for all the users. So as to know the effect of SNR on equivalent noise variance, we plot the first graph. In the first graph we can observe that as the number of quantization levels increase at a given SIR, the equivalent noise variance decreases. This is obvious because as the number of quantization levels increase, the quantization noise decreases and therefore $\sigma_\eta{}^2$ decreases and hence equivalent noise variance decreases. Also equivalent noise variance is a monotonic function of γ .The variations of equivalent noise variance with respect to SIR decreases as the number of quantization levels increase. This is because as the number of quantization levels increase α_E increase and α_A decreases therefore the slope decreases and hence the variations are low.

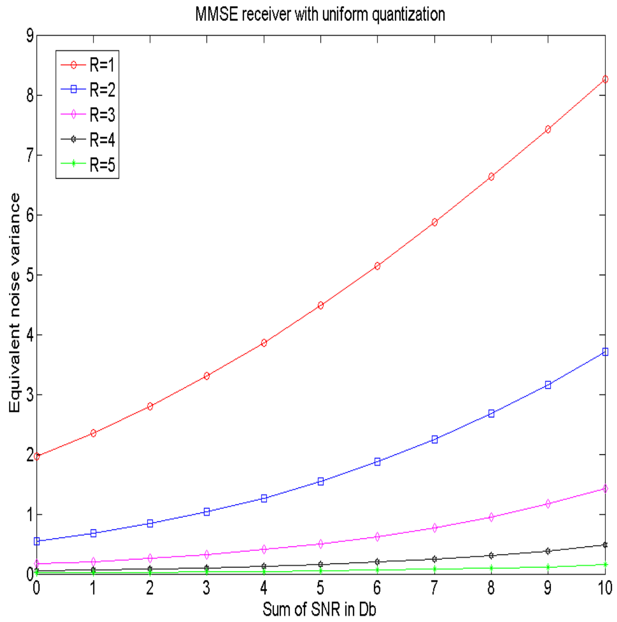

FIGURE 2. The normalized equivalent noise variance versus the sum of SNR's.

In the second graph we can observe that as the number of quantization levels increase the performance of the quantized system approaches that of unquantized.This is because as the number of quantization levels increase the quantization error reduces and thereby the approximation becomes accurate and therefore the performance of the system approaches that of unquantized.

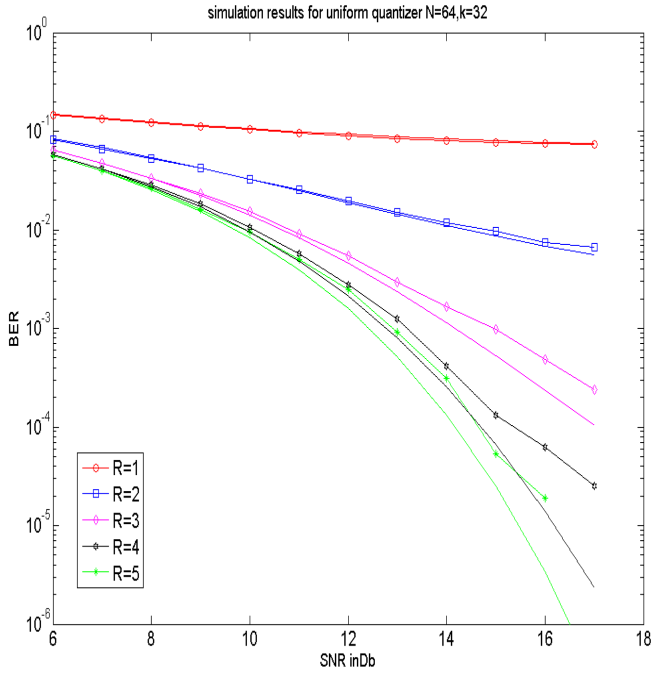

FIGURE 3. BER for the uniformly quantized CDMA signal with MMSE receiver and perfect power control. N=64, K=32.

5 IMPROVEMENT

In the above graphs we have seen that BER curves and equivalent noise variance curves are dependent on the number of quantization levels. We can achieve better results if we could decrease the quantization error for the same value of R. Decreasing the quantization error decreases the covariance of the additive noise vector, which in turn reduces the interference .As the interference reduces the SIR increases which in turn reduces the BER. Therefore reduction in quantization error reduces the BER. We can achieve low quantization errors by adopting non uniform quantization instead of uniform quantization. Therefore we can reduce the BER by using non uniform quantization.

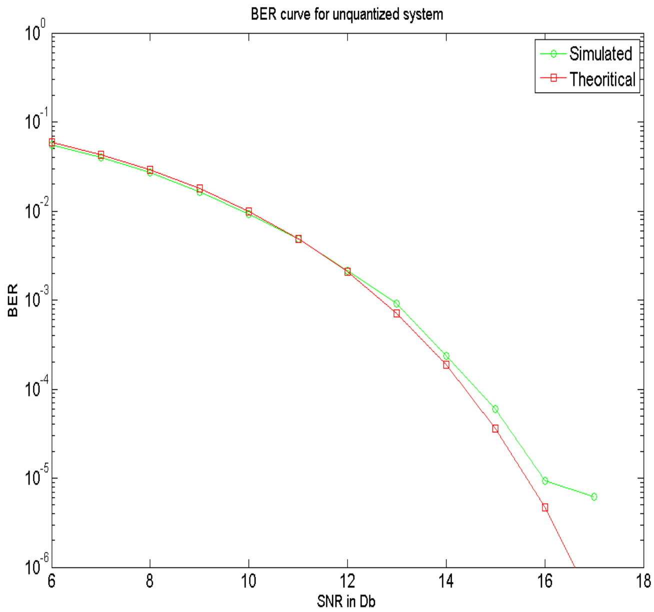

FIGURE 4. BER for the unquantized CDMA signal with MMSE receiver and perfect power control. N=64, K=32.

6 CONCLUSION

The simulation results are computed by transmitting 32* 10^4 bits. We assume that the number of users transmitting synchronous CDMA signals are 32 and that the processing gain is 64.We also assume that all the users receive same power . Simulation results will come much closer to the theoretical results if we increase the number of transmitted bits.

7 RESULT

The SNR of unquantized signal is greater than the uniform quantized signal i.e. ,the BER of unquantized signal is less than the uniform quantized signal.

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